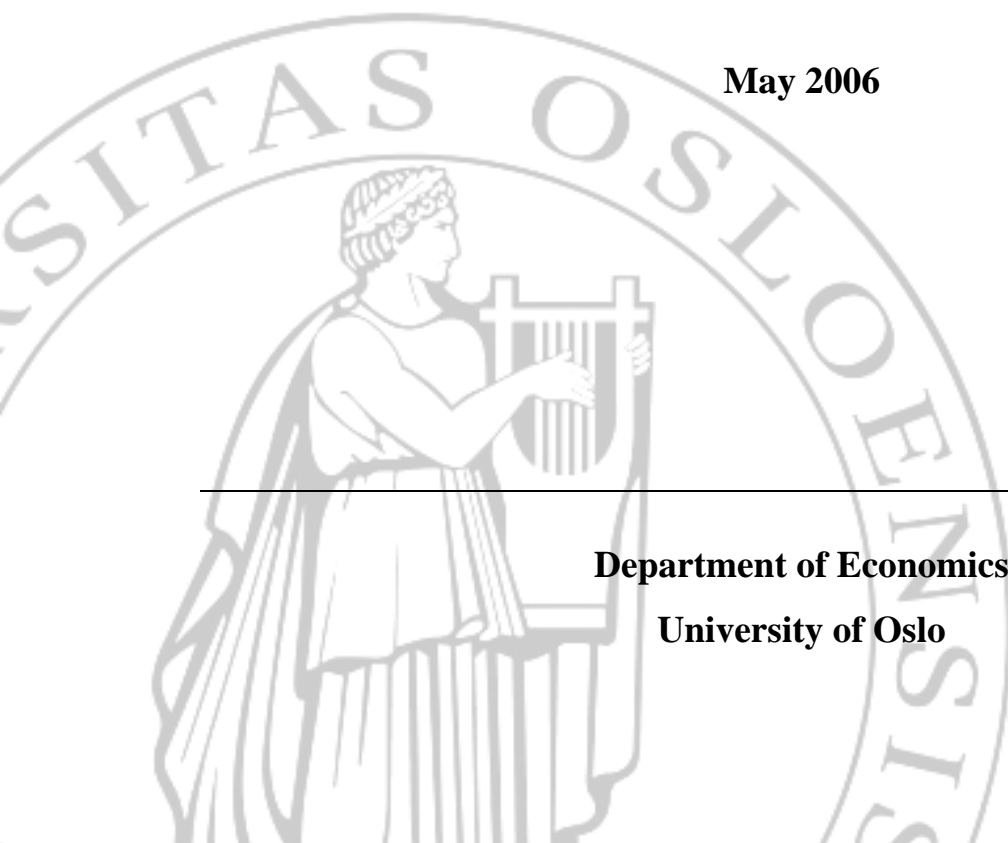


Theoretical Analysis on Income Arising From the Petroleum Sector

Wei, Taoyuan

May 2006

**Department of Economics
University of Oslo**



Preface

Green accounting is my interest. The master thesis on sectoral income arising from the petroleum sector is a good start for me. At the moment of completion, I would in particular like to thank my supervisor Geir B. Asheim for excellent and insightful supervision – and for his attentive, interested, and helpful attitude. Without his enthusiasm and encouragement, this work had never been possible. The Master thesis now becomes one part of my Ph.D programme on green accounting, which is partly supported by the Norwegian Quota Programme as of January 2006.

It is not a short period for me to complete the Master's Degree in University of Oslo. During the process, a number of people were important, and they will not be forgotten. Among them are encouraging and helpful friends, and the cheerful people in my study group at the University of Oslo. Olav Bjerkholt and Solveig Glomsrød deserve to be especially mentioned because they – in their unique ways – contributed to my master study. I am also grateful for the good working environment offered at the Department of Economics, University of Oslo during the study. Finally I would like to thank my wife, Junhui Jiao, and other family members for their supporting, understanding and encouragement.

I confirm here that all the remaining errors are due to my own responsibilities.

Contents

1	Introduction	1
2	The concept of sectoral income	2
3	The petroleum sector: definition, production, and cost	4
4	Sectoral income in a Dasgupta-Heal-Solow model	8
5	Partial models with endogenous price determination	12
5.1	Costless extraction	12
5.1.1	Effects of demand curve movement	15
5.1.2	Effects of the real interest rate change	17
5.2	Constant unit extraction cost	18
5.3	Constant but different unit extraction costs	21
5.4	Stock dependent cost	23
5.5	Technological improvement	27
5.6	Technological improvement and stock dependent cost	29
5.7	Effects of time dependent real interest rate	31
6	Partial models with exogenous price determination	32
6.1	Constant unit extraction cost	32
6.2	Variable extraction costs w.r.t. extraction alone	33
6.3	Stock dependent cost	36
6.4	Exogenously determined extraction	38
6.4.1	Constant unit extraction cost	38
6.4.2	Variable cost w.r.t. extraction alone	40
6.4.3	Stock dependent cost	42
7	Concluding remarks	44

1 Introduction

Recently, following the concept of income presented by Sefton and Weale (2005) as well as earlier contributions, a new theoretical framework on sectoral income in comprehensive national accounting has been developed by Asheim and Wei (2006). The present thesis aims to apply this framework to non-renewable resource sectors, in particular the petroleum sector. In the thesis, the income arising from the petroleum sector are discussed in cases of various theoretical resource models. The approach can also be used to analyze the income of other exhaustible resource sectors.

Sectoral income is defined in the paper as the present value of real interests on current and all the future real cash flow arising from a given sector. Or alternatively, the sectoral income can be split into its current cash flow, its net investments, and its price change effects.

In the petroleum sector, the real cash flow at each point in time includes two kinds of resource rents, namely the Hotelling rent and Ricardian rent. In general, the Hotelling rent is defined as the difference between the market price of the resource and marginal extraction costs (Hotelling, 1931). When an input factor consists of heterogeneous units, the Ricardian rent might be seized on the units with higher productivity if the less productive units are called into the production (Ricardo, 1821). The Ricardian rent in the petroleum sector is generated due to the heterogeneous productivity of ground input within each point in time.

Through applying the concept of income in various theoretical petroleum models, the paper determines and classifies the income arising from the petroleum sector. The main difference between the mentioned models is the form of extraction cost functions, from functions with costless extraction to functions where cost depends on the petroleum stock and technological improvement. In some cases, the sectoral income comes from the petroleum price change effects. In some other cases, however, it comes from the Ricardian rent, technological improvement, or even from the change of real interest rate.

The organization of this thesis is straightforward. Next section introduces in brief the concept of sectoral income. This is followed by a discussion of what is meant by the petroleum sector in this paper. Then an analysis is illustrated by calculating the resource

income in a Dasgupta-Heal-Solow model. In the following sections the expressions for income arising from the petroleum sector are discussed in various partial models. Firstly the partial models with endogenous price determination are analyzed. Secondly the partial models with exogenous price determination are discussed. The final section concludes the main findings of this thesis.

2 The concept of sectoral income

Hicks (1939) defined income as "the maximum amount a man can spend and still be as well off at the end of the week as at the beginning". As Sefton and Weale (2005) argue, if "as well off" is understood to mean that "the present discounted value of current and future utility should be unchanged over the interval considered", then the income can be defined as a weighted average of current and future consumption flows. Following Samuelson (1961), welfare improvement is measured by the present value of future changes in consumption. By defining this to be savings and adding it to present consumption, we obtain the concept of income defined in Sefton and Weale (2005). Furthermore, by dividing this in a consistent manner into different sectors, Asheim and Wei (2006) present a definition of sectoral income, which can be expressed in two alternative ways. One approach comes from the real cash flow to a given sector over time. The income from a sector j at current time $t = 0$ can be expressed by

$$Y_0^j = \int_0^\infty (-\dot{\pi}_t^j) \mathbf{P}_t^j \mathbf{x}_t^j dt = \int_0^\infty R_t^j \pi_t^j \mathbf{P}_t^j \mathbf{x}_t^j dt. \quad (1)$$

Where, π_t^j is a Divisia consumer price index for sector j from its path of real consumption interest rates $\{R_t^j\}_{t=0}^\infty$,

$$\pi_t^j = e^{-\int_0^t R_\tau^j d\tau}$$

for all $t \geq 0$. If it is assumed that there is a constant real interest rate R , then

$$\pi_t^j = e^{-Rt}.$$

Clearly we have that $\pi_0^j = 1$ and

$$R_t^j = -\dot{\pi}_t^j / \pi_t^j$$

for all $t \geq 0$. $\{\mathbf{P}_t^j\}_{t=0}^\infty$ is the path of market (or calculated) *real* commodity prices for sector j derived from the path of market present value prices $\{p_t^j\}_{t=0}^\infty$,

$$\mathbf{P}_t^j = \mathbf{p}_t^j / \pi_t^j$$

for all $t \geq 0$. And $\{\mathbf{x}_t^j\}_{t=0}^\infty$ is the path of sector j 's vector of commodity flows. In the following context, we denote the market real prices by capital letters and the market present value prices by small letters.

For each sector j , R_t^j is an average real consumption interest rate to be used for the calculation of sector j 's income. To make it simple, in the following analysis, we assume that R_t^j coincides with the real interest rate at the national level as R_t . Then the definition (1) can be rewritten as

$$Y_0^j = \int_0^\infty (-\dot{\pi}_t) \mathbf{P}_t^j \mathbf{x}_t^j dt = \int_0^\infty R_t \pi_t \mathbf{P}_t^j \mathbf{x}_t^j dt \quad (2)$$

On the other hand, the differentiation of $\pi_t \mathbf{P}_t^j \mathbf{x}_t^j$ yields

$$\frac{d}{dt}(\pi_t \mathbf{P}_t^j \mathbf{x}_t^j) = \dot{\pi}_t \mathbf{P}_t^j \mathbf{x}_t^j + \pi_t \dot{\mathbf{P}}_t^j \mathbf{x}_t^j + \pi_t \mathbf{P}_t^j \dot{\mathbf{x}}_t^j.$$

Integrating on both sides under the assumption that $\pi_t \mathbf{P}_t^j \mathbf{x}_t^j \rightarrow 0$ as $t \rightarrow \infty$, leads to the following equation:

$$-\mathbf{P}_0^j \mathbf{x}_0^j = \int_0^\infty \dot{\pi}_t \mathbf{P}_t^j \mathbf{x}_t^j dt + \int_0^\infty \pi_t \dot{\mathbf{P}}_t^j \mathbf{x}_t^j dt + \int_0^\infty \pi_t \mathbf{P}_t^j \dot{\mathbf{x}}_t^j dt.$$

By rearranging this equality and applying expression (2) we obtain

$$Y_0^j = \underbrace{\mathbf{P}_0^j \mathbf{x}_0^j}_{\text{current cash flow}} + \underbrace{\int_0^\infty \pi_t \mathbf{P}_t^j \dot{\mathbf{x}}_t^j dt}_{\text{sectorial net investments}} + \underbrace{\int_0^\infty \pi_t \dot{\mathbf{P}}_t^j \mathbf{x}_t^j dt}_{\text{price change effects}}. \quad (3)$$

Hence, for each sector j , we are able to split the sector's income into its current cash

flow, its net investments, and its price change effects.

For comparison, we also write down the expression of income as interests on wealth following Usher (1994), where the wealth of one sector is defined as the present discounted value of its future real cash flow that could be used for consumption. Following the notation here, income as interests on wealth can be expressed as

$$YW_0^j = R_0 \int_0^\infty \pi_t \mathbf{P}_t^j \mathbf{x}_t^j dt. \quad (4)$$

The difference between the two measures in expressions (2) and (4) comes from the change of real interest rate over time. If the real interest rate keeps constant over time, the results derived from the two measures are exactly the same. However, as the real interest rate falls over time, the income calculated from expression (2) must be less than that as interests on wealth. Sefton and Weale (2005) give more details on this comparison.

3 The petroleum sector: definition, production, and cost

For simplicity, the petroleum sector here is defined as all the petroleum reservoirs where the underground petroleum can be used as input in current or future extraction activities within an economy. Accordingly, income arising from the petroleum sector is the sum of income arising from the extraction of the petroleum resource. There are undoubtedly many firms operating on the extraction. Each firm is assumed to own one reservoir so that each firm's income share for the petroleum resource can be thought of as each reservoir's income. If we can determine the resource income of each firm in the sector, the sectoral income then is just the sum of the resource income earned by firms. Therefore, we emphasize on the resource income of each firm (or reservoir) in the petroleum extraction activities.

In the following sections, we always assume that one reservoir has given initial resource stock (S_0). The reservoir is also assumed to be exhausted after the extraction lasts for a certain period. if let T denote the time at which the reservoir is exhausted, then there are relations between the extraction rate over time (r_t) and the remaining petroleum stock

at time m (S_m)

$$\int_m^T r_t dt = S_m \quad (5)$$

for all $m \geq 0$.

In the analysis of the petroleum sector, all the petroleum products extracted directly from the underground are assumed to be homogenous.

Next we turn to production and cost of the petroleum sector.

In this thesis, we mainly concentrate on three factors in the process of the petroleum extraction.

First of all, the effort to extract petroleum is abstracted as one variable, a . The effort is a composite input (for example, labor and capital) for drilling, pumping, and, where appropriate, injection during the extraction process. The real market price of the effort a is denoted by W , which, in the following analysis, is assumed to keep constant over time.

Then the remaining stock S_t are considered. Often the extraction rate tends to go down as the remaining stock declines if the production technology and the extraction effort is given. This is led by the geologic structure change in a petroleum reservoir. Given the initial stock S_0 , more cumulated extraction implies less remaining stock, which corresponds to a situation where the geologic structure changes so that it becomes more difficult to extract one more unit of the petroleum. Then the remaining stock, corresponding to a certain situation of the geologic structure in a reservoir, can be thought of as the quantified situation of the geologic structure. Therefore, if we abbreviate "the situation of the geologic structure" as "the ground", then the remaining stock in the production or cost functions actually stands for the quantified ground input (G). Consequently, the productivity corresponding to the remain stock in the production process represents the productivity of the ground input. In other words, the ground input is qualitatively different from one unit extraction to another and thus has different productivity. This is quantified via the corresponding remaining stock.

The third factor we considered here is the technological improvement. This factor tends to improve the total factor productivity and alleviate the increasing marginal cost over time. In the sectoral level, the technological improvement is thought of as exogenously determined and the individual producer can only accept the technology and has

no effects on it at each point in time. The production or cost functions in this thesis are designed so that they do not depend directly on the time variable t . The technological improvement over time is assumed to be an increasing variable (z_t) with respect to the time t . Then the variable z_t appears in the production and cost functions. Thus, the functions have the same forms over time. As discussed later, the effect of technological improvement on sectoral income can be thought of as an adjustment to a price of extraction services.

In order to understand and explain better later, the dual relations between the cost and the production functions are presented below. Since the function forms are the same over time, in the following of this section we ignore the subscript t for the variables.

The production function is supposed to capture the characteristics of the petroleum extraction process. As we mentioned above, three factors are considered in the function,

$$r = F(a, S, z) . \quad (6)$$

If the production has the property of constant returns to scale (CRS) with respect to inputs a and S (the technological variable z is exogenously determined and the producer can not adjust it), then we rewrite the production function as

$$r = F(a, S) , \quad (7)$$

and for any given positive real number μ , we know

$$y = \mu r = F(\mu a, \mu S) . \quad (8)$$

The differentiation with respect to μ gives that

$$\frac{dy}{d\mu} = r = F'_a a + F'_S S, \quad (9)$$

where we have used the abbreviation F'_a and F'_S for $F'_a(a, S)$ and $F'_S(a, S)$. We also use similar abbreviations for other terms in the following context. Equation (9) can be interpreted as that production equals the total productivity of both the effort and the

ground inputs. The productivity of the ground input is represented by the second term in the right hand of expression (9).

Notice that expression (9) has an implicit assumption as changing ground input quality (or the remaining stock S) within the point in time. Otherwise, if ground input quality is constant, then its representative, the remaining stock S , is constant and can be ignored from the production function (7). If so, the property of CRS can not exist as the form in expression (9). For example, if we assume there is only one unit ground input (constant over time), we have no way to explain the changing productivity of the same unit ground input along with increasing extraction rate within each point in time as long as the property of CRS in (9) holds.

At the same time, the property of CRS does not rely on the initial ground input quality (or the remaining stock S) at the beginning of each point in time even though the level of productivity of ground input at each point in time does depend on the initial ground input quality.

On the opposite side of the assumption required by (9), if within each point in time the remaining stock S and technological variable z are constant, the production function can be simply rewritten as

$$r = F(a), \quad (10)$$

as long as there are some stock $S > 0$. However, this does not mean that the remaining stock S or technological variable z has no contributions to the production. The simplified production function has the implication as

$$dS = 0, \quad dz = 0. \quad (11)$$

Totally differentiate both hands of the production function (6),

$$\begin{aligned} dr &= dF = F'_a da + F'_S dS + F'_z dz \\ &= F'_a da \quad \text{by (11)} \end{aligned} \quad (12)$$

or

$$da = \frac{dr}{F'_a} \quad (13)$$

Then we know the marginal product with respect to a as that

$$\frac{dr}{da} = \frac{dF}{da} = F'_a . \quad (14)$$

The cost function can be found out by minimizing the cost of the effort input subject to the production relations and the exogenously given real market price of the effort (W). This can be written as that

$$C(r, S, z) = Wa = Wa(r, S, z) , \quad (15)$$

here the effort input a has to be derived from the production function with given r , S , and z .

Similarly the cost function can be simplified as

$$C(r) = Wa(r). \quad (16)$$

The differentiation of cost function (16) gives

$$dC = Wda(r) \quad (17)$$

By using the result in (13), expression (17) can be rewritten as

$$dC = W \frac{dr}{F'_a} \quad (18)$$

The marginal cost can be easily got as

$$C'_r = \frac{dC}{dr} = \frac{W}{F'_a} . \quad (19)$$

By equations (14) and (19), the marginal product derived from production function and the marginal cost derived from the cost function can be easily transposed from one

to another with knowledge of the exogenously given market price of the efforts (W).

4 Sectoral income in a Dasgupta-Heal-Solow model

This section follows the description in Asheim and Wei (2006).

Consider the Cobb-Douglas Dasgupta-Heal-Solow model (Dasgupta and Heal, 1974, 1979; Solow, 1974). Hence, production, q_t at time t is given by

$$q_t = k_t^\alpha r_t^\beta$$

where k is the capital stock, r is resource input being extracted at no cost from a finite stock, and the available labor ℓ is constant and normalized to one (i.e. $\ell_t = 1$ for all t), and where we assume that

$$1 > \alpha + \beta > \alpha > \beta.$$

Production can be split into consumption c_t and accumulation of capital \dot{k}_t :

$$q_t = c_t + \dot{k}_t.$$

Since this is a one-consumption good model, price indices need not be invoked. Consequently, the real price of consumption can be set to 1 for all $t \geq 0$. The real price of resource input P_t , and the real interest rate R_t equals the marginal productivity of inputs:

$$P_t = \beta q_t / r_t,$$

$$R_t = \alpha q_t / k_t.$$

Furthermore, along an efficient path, the Hotelling rule,

$$\pi_t P_t = P_0,$$

is satisfied, where $\{\pi_t\}_{t=0}^\infty$ is the path of present value prices of consumption:

$$\pi_t = e^{-\int_0^t R_\tau d\tau}$$

for all $t \geq 0$.

Assume that the economy follows the efficient constant consumption path, which exists under these assumptions. This path is characterized by a constant production q , with the constant consumption being a fixed share of production: $c = (1 - \beta)q$, and with the reminder being used for capital accumulation:

$$\dot{k}_t = \beta q.$$

Consider the sector corresponding to the supply of resource input, which only provides resource input and is not responsible for capital accumulation. The real cash flow to the sector at each point in time is as follows:

$$P_t r_t = \beta q.$$

Note that

$$\int_0^\infty R_t \pi_t dt = \int_0^\infty (-\dot{\pi}_t) dt = \pi_0 = 1, \quad (20)$$

provided that $\pi_t \rightarrow 0$ as $t \rightarrow \infty$. This implies that sectorial income at time 0 is given by

$$Y_0 = \int_0^\infty R_t \pi_t \beta q dt = \beta q \quad (21)$$

Note that since the real interest rate is decreasing over time, the resource income is lower than the interests on the resource wealth, which can be shown to equal¹

$$R_0 \int_0^\infty \pi_t \beta q dt = \frac{\alpha}{\alpha - \beta} \beta q.$$

However, if the real interest rates were constant over time (something it is not in the Dasgupta-Heal-Solow model along a constant consumption path), then the estimates of the two measures would have been exactly the same.

Alternatively, we can use expression (3) to derive the expression for the resource

¹ $R_\tau = \alpha q / k_\tau = \alpha q / (k_0 + \beta q \tau) = \alpha q / (\alpha q / R_0 + \beta q \tau) = R_0 / (1 + (\beta/\alpha) R_0 \tau)$
 $\implies \int_0^t R_\tau d\tau = (\alpha/\beta) \ln(1 + (\beta/\alpha) R_0 t) \implies \pi_t = (1 + (\beta/\alpha) R_0 t)^{-(\alpha/\beta)}$
 $\implies \int_0^\infty \pi_t dt = -\frac{(\alpha/\beta)^{\alpha/\beta}}{(\alpha/\beta - 1) R_0 (\alpha/\beta + R_0 t)^{\alpha/\beta - 1}} \Big|_0^\infty = \frac{\alpha}{(\alpha - \beta) R_0}$ since $\alpha > \beta$

income.

$$\begin{aligned}
Y_0 &= P_0 r_0 + \int_0^\infty \pi_t P_t \dot{r}_t dt + \int_0^\infty \pi_t \dot{P}_t r_t dt \\
&= P_0 r_0 + P_0 \int_0^\infty \dot{r}_t dt + \int_0^\infty \pi_t \dot{P}_t r_t dt \quad \text{by Hotelling's rule,} \\
&= \int_0^\infty \pi_t \dot{P}_t r_t dt \quad \text{since } \lim_{t \rightarrow \infty} r_t = 0 \text{ implies } \int_0^\infty \dot{r}_t dt = -r(0), \\
&= \int_0^\infty (-\dot{\pi}_t) P_t r_t dt \quad \text{since } \dot{\pi} P + \pi \dot{P} = 0 \text{ by Hotelling's rule,} \\
&= \beta q \quad \text{by (20) since } P_t r_t = \beta q \text{ for all } t.
\end{aligned} \tag{22}$$

This means that the resource income can be split like this:

$$Y_0 = \underbrace{\beta q}_{\text{current cash flow}} - \underbrace{\beta q}_{\text{net investments}} + \underbrace{\beta q}_{\text{price change effects}}, \tag{23}$$

where the negative net investments equal the Hotelling rents and cancel out the value of production. Hence, the resource income can be interpreted as arising from the price change effects.

Then if the term of net investments is thought of as the depletion at each point in time, then for an exhaustible resource owner with zero extraction costs, the depletion might be less or more than the current cash flow at the same time point since it now depends not only on the current extraction but also on the path of market real prices of the resource and the Divisia consumer price index. In particular, in the case of Dasgupta-Heal-Solow model described here, the depletion happens to cancel off the current cash flow, the sectoral income then only consists of the price change effects.

Obviously the Dasgupta-Heal-Solow model is for a whole closed economy. The resource sector is defined as the resource owner who provides the resource after extraction without any costs. Next we will discuss the sectoral income in partial models that focus on the resource sector and neglect the rest of the economy.

5 Partial models with endogenous price determination

In this section, the market petroleum real price is endogenous and depends on the rate of extraction alone. The sector as a whole can affect the equilibrium real price level by adjusting its amount of extraction rate. Here the market real price is denoted by $P(r_t)$. In the next section, the petroleum real price path over time is assumed to be determined exogenously and independent on the rate of extraction. There the petroleum price will be denoted P_t .

5.1 Costless extraction

First of all, we consider the cases with a competitive market in the petroleum sector.

The competitive firms that comprise the petroleum sector are analytically identical. Each firm's extraction problem is to maximize its total present value of the net profits arising from the extraction from a homogeneous petroleum reservoir with known stock at the beginning. Extraction and other costs are neglected. The reservoir is presumed to be one of many in the competitive sector. The sector is in equilibrium when each firm is indifferent to holding or selling one more unit of the extraction. Then it is natural to seek conditions such that the maximizing behavior of firms will generate an equilibrium extraction path for the sector. Since each firm behaves as a price-taker, the price at each point in time is taken as given when the extraction decisions are made even though the price depends negatively on the total extraction amount being sold. Thus, for the sector, the equilibrium path will be obtained by solving the maximization problem with respect to the extraction at each point in time.

$$\begin{aligned} \max_{r_t} \int_0^\infty P(r_t) r_t e^{-Rt} dt \\ \text{s.t. } \dot{S}_t = -r_t \\ S_0 \text{ is given} \end{aligned} \tag{24}$$

Here all the extraction r_t is sold on the market. $P(r_t)$ is the real price in equilibrium in the competitive market at time t . R is the constant real interest rate over time.

The state variable in this problem is the amount of remaining resource S_t , while the control variable is the extraction r_t . The Hamiltonian expression for this problem is

$$H(t, S_t, r_t, \lambda_t) = P(r_t)r_t e^{-Rt} - \lambda_t r_t \quad (25)$$

where λ_t is the present value resource price (shadow price, or co-state variable).

The maximum principle gives conditions (note that the price is thought of as given by each firm)

$$\frac{\partial H}{\partial r_t} = P(r_t)e^{-Rt} - \lambda_t = 0 \quad (26)$$

Furthermore, the rate of change of the adjointed price is given by

$$\dot{\lambda}_t = -\frac{\partial H}{\partial S_t} = 0 \quad (27)$$

Conditions (26) and (27) then imply $P(r_t)e^{-Rt} = \lambda_t = \text{constant}$. Notice that it is satisfied even if $t = 0$, then the condition can be rewritten as

$$P(r_t)e^{-Rt} = P_0 \quad \text{or} \quad \dot{P}(r_t) = RP(r_t) . \quad (28)$$

This is the Hotelling rule in its purest form. Then the equilibrium extraction path in the sector requires that the underground petroleum earns a rate of return equal to R , the same amount as other assets. The Hotelling rule also means that each unit of the extraction earns the same present value of its net price, whenever it is extracted. This makes sense since otherwise all the petroleum will be extracted at the time point with the highest present value of the profit. This denotes that each firm is indifferent between zero output and production at some maximum extraction rate at each point in time.

Let us consider a familiar downward-sloping linear form for the demand curve that is applied to characterize the market demand at each point in time.

$$P(r_t) = \bar{P} - br_t \quad (29)$$

Here \bar{P} is the vertical intercept point (where r is zero) and can be thought of as the

"choke price". The curve has constant slope equal to $(-b)$, where $b > 0$.

The petroleum sector can make the Hotelling rule prevail by adjusting its extraction amount at each point in time (r_t) . At the sectoral level, the equilibrium extraction path is also required to satisfy the demand function (29). Assume T is the time at which the petroleum stock is exhausted, which implies $r_T \rightarrow 0$. Insert conditions (28) into demand function (29) the sector can find the equilibrium current price $P(r_0)$ as

$$\begin{aligned} P_T &= P(r_0)e^{RT} \rightarrow \bar{P} \\ \implies P(r_0) &\rightarrow \bar{P}e^{-RT}. \end{aligned} \quad (30)$$

In the limit sense, $P(r_0) = \bar{P}e^{-RT}$ will be chosen by the sector. Then, all prices are functions of T and so do all the extractions over time. And T can be determined by applying the resource stock constraint condition (5) at time 0. Following this way, in this special case, T can be solved out by the following equation

$$\frac{bS_0}{\bar{P}}R + 1 - RT - e^{-RT} = 0 \quad (31)$$

It is sufficient to assume that the market functions as if the firms are able to make these calculations. Thus, by adjusting its extraction amount to realize the equilibrium sectoral extraction path, each firm can earn the maximum of the present value from its reservoir.

If so, the real cash flow to the petroleum sector over time is $\{P(r_t)r_t\}_{t=0}^{\infty}$. By inserting the Hotelling rule (28) and using condition (5) into expression (2), the income arising from the petroleum sector is obtained as that

$$Y_0 = \int_0^T Re^{-Rt}P(r_t)r_t dt = RP_0S_0 \quad (32)$$

Substitute $P(r_t)$ with $\bar{P}e^{-RT}$, the income can be rewritten as that

$$Y_0 = R\bar{P}e^{-RT}S_0 \quad (33)$$

Along with the equilibrium extraction path in the sectoral level, the income of the

petroleum sector is interests on the present value of the reservoirs, which depends on the real interest rate, total initial petroleum stock in the sector and the demand curve in the market.

Alternatively, we can use expression (3) to derive the expression for the income arising from the petroleum sector.

$$\begin{aligned}
Y_0 &= P(r_0)r_0 + \int_0^\infty e^{-Rt} P(r_t) \dot{r}_t dt + \int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt \\
&= P(r_0)r_0 + P(r_0) \int_0^T \dot{r}_t dt + \int_0^T e^{-Rt} \dot{P}(r_t) r_t dt \quad \text{by Hotelling's rule,} \\
&= \int_0^T e^{-Rt} \dot{P}(r_t) r_t dt \quad \text{since } \lim_{t \rightarrow \infty} r_t = 0 \text{ implies } \int_0^\infty \dot{r}_t dt = -r(0), \\
&= \int_0^T R e^{-Rt} P(r_t) r_t dt \quad \text{by Hotelling's rule as (28),} \\
&= R P_0 S_0 \quad \text{by (32).}
\end{aligned} \tag{34}$$

This means that the resource income can be split like this:

$$Y_0 = \underbrace{P(r_0)r_0}_{\text{current cash flow}} - \underbrace{P(r_0)r_0}_{\text{net investments}} + \underbrace{R P_0 S_0}_{\text{price change effects}}, \tag{35}$$

where the negative net investments equal the Hotelling rents and cancel out the value of production. Hence, the resource income can be interpreted as arising from the price change effects.

5.1.1 Effects of demand curve movement

In this simple model, the demand curve depends on two parameters, the choke price \bar{P} and the constant slope $(-b)$. The movement of \bar{P} implies that the demand curve moves upward or downward. The change of the slope implies that the demand curve rotates around the chock price \bar{P} .

First consider the movement of \bar{P} .

Totally differentiate equation (31) with respect to \bar{P} and T , we obtain that

$$\frac{dT}{d\bar{P}} = -\frac{bS_0}{\bar{P}^2(1 - e^{-RT})} < 0. \tag{36}$$

It is easy to notice that the right hand of equation (36) is negative. The time at which the resource is exhausted, T , moves to the opposite direction of the movement of the choke price \bar{P} .

With more demand for the petroleum at given prices, which corresponds to the upward movement of the demand curve, firms tend to extract more in a given time period and the resource is exhausted a bit earlier (T becomes smaller). Then due to the positive effects of price movement and the corresponding extraction period change, the sectoral income tends to increase, which can be observed from equation (33). On the contrary, less demand for the petroleum at given prices implies the decreasing trend of the sectoral income.

Next consider the change of the constant slope $(-b)$.

Totally differentiate equation (31) with respect to b and T , we obtain that

$$\frac{dT}{db} = \frac{S_0}{\bar{P}(1 - e^{-RT})} > 0 . \quad (37)$$

Notice that the right hand of this equation (37) is positive. The time at which the resource is exhausted, T , moves to the same direction as the change of the absolute value of the slope b .

Given the choke price \bar{P} , if the demand curve becomes steeper, which also implies less demand at a given price other than \bar{P} , firms tend to extract less in a given time period and the resource is exhausted a bit later (T becomes larger). Then the sectoral income tends to decrease, which can be observed from equation (33). On the contrary, flatter demand curve (or more demand for the petroleum at given prices other than \bar{P}) implies increasing sectoral income.

Therefore, in general more demand for the petroleum at given prices leads to shorter extraction period and more sectoral income. Vice versa. The results are easy to understand since the sectoral income will increase when demand increases even though the initial equilibrium extraction path does not change.

5.1.2 Effects of the real interest rate change

Notice that both the real interest rate R and the time at which the resource is exhausted T appear in the expression for the sectoral income (33). The effects of the real interest rate change on the sectoral income then depends on the changes of both R and T .

Totally differentiate equation (31) with respect to R and T , we obtain that

$$\frac{dT}{dR} = \frac{\frac{bS_0}{\bar{P}} - T + Re^{-RT}}{R(1 - e^{-RT})} . \quad (38)$$

By integrating both sides of equation (29) with respect to t , we obtain that

$$\begin{aligned} \int_0^T P_t dt &= \int_0^T \bar{P} dt - b \int_0^T r_t dt \\ &\iff \int_0^T \bar{P} e^{-R(T-t)} dt = \bar{P}T - bS_0 \\ &\iff \frac{1}{R} \bar{P} (1 - e^{-RT}) = \bar{P}T - bS_0 \\ &\iff \frac{bS_0}{\bar{P}} - T = -\frac{1}{R} (1 - e^{-RT}) \end{aligned} \quad (39)$$

Insert (39) into (38) and rearrange, we obtain that

$$\frac{dT}{dR} = \frac{-1 + (1 + R^2)e^{-RT}}{R^2(1 - e^{-RT})} = -\frac{1}{R^2} + \frac{e^{-RT}}{1 - e^{-RT}} \quad (40)$$

It is unclear what will happen on T if R changes since the sign of (40) is undetermined. However, expression (40) can be rewritten by introducing $P_0 = \bar{P}e^{-RT}$ as

$$\frac{dT}{dR} = -\frac{1}{R^2} + \frac{P_0}{\bar{P} - P_0} = -\frac{1}{R^2} + \frac{1}{\bar{P}/P_0 - 1} \quad (41)$$

By expression (41), we can split the effect of R on T into two terms: direct effect and indirect effect via initial price P_0 . Obviously as the direct effect on T of R , a smaller interest rate extends the extraction period. On the other hand, as the indirect effect via P_0 , a smaller interest rate shortens the extraction period by increasing the initial price.

Now we totally differentiate equation (33) with respect to Y_0 , R , and T and apply

expression (40),

$$\begin{aligned}
dY_0 &= (\bar{P}e^{-RT}S_0 - RT\bar{P}e^{-RT}S_0)dR - R^2\bar{P}e^{-RT}S_0dT \\
&= \bar{P}e^{-RT}S_0 \left[1 - RT - R^2 \left(-\frac{1}{R^2} + \frac{e^{-RT}}{1 - e^{-RT}} \right) \right] dR \\
&= \bar{P}e^{-RT}S_0 \left(2 - RT - R^2 \frac{1}{e^{RT} - 1} \right) dR \\
&= P_0S_0 \left(2 - RT - \frac{R^2}{\bar{P}/P_0 - 1} \right) dR
\end{aligned} \tag{42}$$

By equation (42), we still can not make sure the effects of R change on the sectoral income. In this case, there are three effects of the real interest rate change. One is the direct effect of R . A smaller interest rate has a direct negative effect since R appears as the first factor in the right hand of expressions (32) and (33). Another one is the price change effect. A smaller interest rate increases the initial petroleum price $P_0 = \bar{P}e^{-RT}$ (given constant extraction period T); this is the positive effect. The last one is the extraction period effect. This effect is undetermined since the change of the extraction period is unclear as we mentioned above. But the extraction period effect of R comes from the direct effect of R and the indirect effect of P_0 . Thus, we can thought of the effects of the R change on the sectoral income as two terms: the direct negative effect of R itself and the indirect positive effect of initial price P_0 .

For example, if assume $R = 0.07, T = 20$, Then the sectoral income will move on the same direction as the change of R . However, if assume $R = 0.07, T = 30$, Then the sectoral income will move on the opposite direction of the change of R .

As a result, in this ideal case, the effects of interest rate changes on the sectoral income are unclear.

5.2 Constant unit extraction cost

This part assumes the constant unit extraction cost. Each unit of the extraction has the same cost, which also means the marginal cost (MC) is constant any time.

$$C(r_t) = cr_t, \quad c \text{ is a positive constant.}$$

Given the initial stock S_0 , these costs can be thought of as the allocation share on each unit extraction of fixed extraction cost, pre-extraction cost, or exploration cost. These costs are thought of as exogenously constant. At current step, we neglect the effects of the remaining stock (or the accumulative extraction) and extractive technological improvement that occur as time goes on.

If the demand curve is given as the same linear form in the previous section 5.1, it is also easy to include these costs into the model that have just been displayed in the section 5.1. Since these costs do not rely on the current stock or extraction rate. The profit are now total revenues minus costs. For the sector, the equilibrium path now can be obtained by solving the maximization problem as follows.

$$\begin{aligned} \max_{r_t} \int_0^\infty (P(r_t)r_t - cr_t)e^{-Rt}dt \\ \text{s.t. } \dot{S}_t = -r_t \\ S_0 \text{ is given} \end{aligned} \quad (43)$$

It is obvious that we can do the same analysis as section 5.1 just by replace $P(r_t)$ in the problem (24) with $(P(r_t) - c)$. Notice that in this case we must have $\bar{P} > c$. Otherwise the extraction will never occur. Then we know the sectoral income has the similar form as that in expression (32).

$$Y_0 = \int_0^T Re^{-Rt}(P(r_t) - c)r_t dt = R(P(r_0) - c)S_0 \quad (44)$$

So far, the firms is assumed to be competitive such that no one firm can affect the market price by increasing or decreasing its extraction.

Now let us move to consider the *monopoly case*, where the petroleum market is dominated by a single firm. The monopolist also faces a given declining demand curve but has the power to affect the price by adjusting its extraction amount. The demand curve as a function of the extraction rate is supposed to be the same at each point in time. In this case, to make it more practical, we introduce the constant unit cost for the extraction as

c. Then the problem for the monopolist is to

$$\begin{aligned} \max_{r_t} \int_0^\infty (P(r_t)r_t - cr_t)e^{-Rt}dt \\ \text{s.t. } \dot{S}_t = -r_t \\ S_0 \text{ is given} \end{aligned} \quad (45)$$

The Hamiltonian expression for this problem is

$$H(t, S_t, r_t, \lambda_t) = (P(r_t)r_t - cr_t)e^{-Rt} - \lambda_t r_t \quad (46)$$

where r_t is the control variable and S_t the state variable.

Since the market price in equilibrium depends on the rate of the monopolist's extraction, the conditions given by the maximum principle turn out to be that

$$\frac{\partial H}{\partial r_t} = [P(r_t) - c]e^{-Rt} + r_t P'(r_t)e^{-Rt} - \lambda_t = 0 \quad (47)$$

Furthermore, the rate of change of the adjointed price is given by

$$\dot{\lambda}_t = -\frac{\partial H}{\partial S_t} = 0, \quad (48)$$

which implies $\lambda_t = \lambda = \text{constant}$.

In order to simplify (47), we introduce the elasticity of demand at each point in time as

$$\epsilon_t = \frac{dr_t}{dP(r_t)} \frac{P(r_t)}{r_t} \quad (49)$$

Then it is easy to notice that

$$r_t P'(r_t) = r_t \frac{1}{dr_t/dP(r_t)} = \frac{1}{\epsilon_t} P(r_t) \quad (50)$$

Substitute (50) into (47) and rearrange to get that

$$(1 + \frac{1}{\epsilon_t})P(r_t) - c = \lambda e^{Rt} \quad (51)$$

By letting $t = 0$ in (51), we solve for λ as

$$\lambda = (1 + \frac{1}{\epsilon_0})P(r_t) - c \quad (52)$$

Then by expression (3) the income of the monopolist can be written as that

$$\begin{aligned} Y_0 &= (P(r_0) - c)r_0 + \int_0^T e^{-Rt}(P(r_t) - c)\dot{r}_t dt + \int_0^T e^{-Rt}P'(r_t)\dot{r}_t r_t dt \\ &= (P(r_0) - c)r_0 + \int_0^T e^{-Rt}(P(r_t) - c + \frac{1}{\epsilon_t}P(r_t))\dot{r}_t dt \quad \text{by (50)} \\ &= (P(r_0) - c)r_0 + \int_0^T \lambda \dot{r}_t dt \quad \text{by (51)} \\ &= (P(r_0) - c)r_0 + \lambda r_0 \\ &= 2(P(r_0) - c)r_0 + \frac{1}{\epsilon_0}P(r_0)r_0 \quad \text{by (52)} \end{aligned} \quad (53)$$

Thus it shows that the sectoral income in the monopoly case can be calculated on the basis of current variables. Whether the income is greater than the current cash flow (or profit) or not depends on the current elasticity of demand.

5.3 Constant but different unit extraction costs

In reality the extraction costs tend to increase along with more accumulated extraction. As an intermediate step closer to the practice, here we consider one reservoir with two kinds of stocks, low-cost part and high-cost part. Let S_0 still denote the total initial stock, and S_1 the low-cost part of the stock and S_2 the high-cost part. Further assume the constant low unit extraction cost as c_1 and the high one as c_2 and $c_2 > c_1$.

One can view the high-cost phase as a competitive sectoral equilibrium, which we have explored above, with another competitive sector phase grafted as an earlier phase. It is easy to find out that the maximization behavior of the firm ensures the sequential extraction of the stock. No matter what the form of the declining demand curve is with respect to the extraction rate, the competitive firm will extract if the below relation like (28) is fulfilled.

$$\begin{aligned}
(P(r_t) - c_i)e^{-Rt} &= \text{constant or} \\
\dot{P}(r_t) &= R(P(r_t) - c_i) \quad i = 1, 2
\end{aligned} \tag{54}$$

Obviously we have the following relation since $c_2 > c_1$

$$\frac{\dot{P}(r_t)}{P(r_t) - c_2} > \frac{\dot{P}(r_t)}{P(r_t) - c_1} \tag{55}$$

A rational firm must at first extract the low-cost petroleum since more unit profit can be earned. If so, relation (54) means the right hand of (55) equal to R in the first extraction phase. The high-cost petroleum is then left in the ground to earn a return exceeding the real interest rate R .

Assume at time point T_1 , the low-cost petroleum is exhausted and then the extraction of the high-cost petroleum starts and lasts until the time point T_2 . We also know that the price at T_1 that links the two phases must be the same. Otherwise, consumers will take advantage of the price difference to save the payment by waiting (if there is a downward price) or speeding up consumption (if there is a upward price). This case is described in Hartwick and Olewiler (1998).

Then the sectoral income in the second phase can be estimated by (44) and then discounted to be the present value as

$$Y_{02} = e^{-RT_1} \int_{T_1}^{T_2} Re^{-Rt} (P(r_t) - c_2) r_t dt = Re^{-RT_1} (P_{T_1} - c_2) S_2, \tag{56}$$

And the income from the first phase can be obtained directly by (44)

$$Y_{01} = \int_0^{T_1} Re^{-Rt} (P(r_t) - c_1) r_t dt = R(P(r_0) - c_1) S_1 \tag{57}$$

By replacing P_{T_1} in (56) with $P(r_0)e^{RT_1}$, combine (56) and (57) to find that the total

income of the reservoir is

$$\begin{aligned}
Y_0 &= R(P(r_0) - c_1)S_1 + Re^{-RT_1}(P(r_0)e^{RT_1} - c_2)S_2 \\
&= R(P(r_0) - c_1)S_0 - Re^{-RT_1}(c_2 - c_1)S_2 \quad \text{by (54)}
\end{aligned} \tag{58}$$

When comparing the result with that in (44), we notice that the reduced part of the sectoral income corresponds to the present value of the interests on the extra value of costs estimated at the time point when the low-cost petroleum is exhausted. Thus, any components that extend the length of the first phase without changing the initial price will cause the total sectoral income to increase and Vice versa.

5.4 Stock dependent cost

The extraction cost tends to rise as the remaining stock decline given the constant technologies and the same level of extractive effort. The petroleum resource occurs near the surface of the earth's crust and at various depths and in various degrees of contiguous abundance. Oil pressure is commonly augmented with water and sometimes natural gas, injected into a declining stock during the extraction. Then a usual way to capture such effects is to include the remaining stock S in the production function together with the extractive effort a and so in the cost function together with the extraction rate r . here we concentrate on the cost function.

Assume the extraction cost is a function with respect to not only the extraction r_t , but also the remaining stock S_t . The cost function can be expressed as $C(r_t, S_t)$. In general, $C'_s(R_t, S_t) < 0$ is required.

Still assume the demand curve is linearly declining, the same as described in equation (29).

The reservoir's owner behaviors to find the equilibrium extraction path over time by maximizing the present value of total profit. He faces the following maximization problem.

$$\begin{aligned}
&\max_{r_t} \int_0^\infty [P(r_t)r_t - C(r_t, S_t)] e^{-Rt} dt \\
&\quad s.t. \quad \dot{S}_t = -r_t \\
&\quad S_0 \text{ is given}
\end{aligned}$$

The state variable in this problem is the amount of remaining resource S_t , while the control variable is the extraction r_t . The Hamiltonian of this problem is

$$H(t, S_t, r_t, \lambda_t) = [P(r_t)r_t - C(r_t, S_t)] e^{-Rt} - \lambda_t r_t$$

Then the maximum principle gives conditions

$$\begin{aligned} \frac{\partial H}{\partial r_t} &= [P(r_t) - C'_r(r_t, S_t)] e^{-Rt} - \lambda_t = 0 \\ \dot{\lambda}_t &= -\frac{\partial H}{\partial S_t} = C'_S(r_t, S_t) e^{-Rt} \end{aligned}$$

Differentiate the first condition with respect to time t and using the second condition, we arrive at

$$\left[\dot{P}(r_t) - \dot{C}'_r(r_t, S_t) \right] e^{-Rt} - R [P(r_t) - C'_r(r_t, S_t)] e^{-Rt} = C'_S(r_t, S_t) e^{-Rt}$$

Then we obtain the form of Hotelling's rule in the special cases as

$$\frac{\dot{P}(r_t) - \dot{C}'_r(r_t, S_t)}{P(r_t) - C'_r(r_t, S_t)} = R + \frac{C'_S(r_t, S_t)}{P(r_t) - C'_r(r_t, S_t)} \quad (59)$$

This implies when extraction costs vary negatively with the remaining stock, the Hotelling rent should increase at a rate less than the real interest rate. When compared with the costless cases in section 5.1, it will take a longer time to arrive at the chock price \bar{P} if the initial price $P(r_t)$ stays at the same level.

Next we consider one specific form of such cost functions, which assumes constant unit cost within each point in time t but variable unit cost along with the remaining stock S over time.

The extraction cost function is assumed to have the form

$$C(r_t, S_t) = r_t g(S_t) \quad (60)$$

Often the unit (or marginal) cost of extraction $g(S_t)$ is assumed to be an decreasing function of the remaining stock, but independent of the current extraction r_t . Farzin

(1992, p 820) applied this specific form to show the non-monotonic scarcity rent path of exhaustible resources. The form is consistent with that analyzed by Heal (1976); Hanson (1980); Solow and Wan (1976).

It is easy to notice that

$$\begin{aligned} C'_r(r_t, S_t) &= g(S_t) \\ C'_S(r_t, S_t) &= r_t g'(S_t) = -g'(S_t) \dot{S}_t = -\dot{g}(S_t) \\ \dot{C}'_r(r_t, S_t) &= \dot{g}(S_t) = -C'_S(r_t, S_t) \end{aligned} \tag{61}$$

Insert conditions (61) to equation (59), we obtain

$$R = \frac{\dot{P}(r_t)}{P(r_t) - C'_r(r_t, S_t)} = \frac{\dot{P}(r_t)}{P(r_t) - g(S_t)} \tag{62}$$

This expression (62) shows that interests on the real cash flow exactly equal the petroleum real price changes at each point in time.

Since the real price is assumed to follow the same pattern as the declining linear function in section 5.1, the petroleum sector as a whole can make the equation (62) satisfied through adjusting its amount of extraction rate.

From equation (62) we can derive that

$$\begin{aligned} P(r_t) - g(S_t) &= \frac{\dot{P}(r_t)}{R} \\ \dot{P}(r_t) - \dot{g}(S_t) &= \frac{\ddot{P}(r_t)}{R} \end{aligned} \tag{63}$$

The real cash flow at each point in time

$$P(r_t)r_t - C(r_t, S_t) = [P(r_t) - g(S_t)] r_t$$

By applying expression (2), we can directly express the sectoral income as

$$Y_0 = \int_0^\infty R e^{-Rt} [P(r_t) - g(S_t)] r_t dt,$$

which is the present discounted value of interests on the current and future real cash flow arising from the sector.

Alternatively, since we know

$$\begin{aligned}
& \frac{\partial}{\partial t} [P(r_t) - g(S_t)] r_t \\
&= [P(r_t) - g(S_t)] \dot{r}_t + [\dot{P}(r_t) - \dot{g}(S_t)] r_t \\
&= \frac{1}{R} [\dot{P}(r_t) \dot{r}_t + \ddot{P}(r_t) r_t] \\
&= \frac{1}{R} \frac{\partial}{\partial t} (\dot{P}(r_t) r_t)
\end{aligned} \tag{64}$$

By using expression (3), the sectoral income at current time is then by

$$\begin{aligned}
Y_0 &= [P(r_0) - g(S_0)] r_0 + \int_0^\infty e^{-Rt} [(P(r_t) - g(S_t)) \dot{r}_t - \dot{g}(S_t) r_t] dt + \int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt \\
&= [P(r_0) - g(S_0)] r_0 + \int_0^\infty e^{-Rt} \frac{1}{R} \frac{\partial}{\partial t} (\dot{P}(r_t) r_t) dt \quad \text{by (64)} \\
&= [P(r_0) - g(S_0)] r_0 + \frac{1}{R} \left[e^{-Rt} \dot{P}(r_t) r_t \Big|_0^\infty - \int_0^\infty (-R) e^{-Rt} \dot{P}(r_t) r_t dt \right] \\
&= [P(r_0) - g(S_0)] r_0 - \frac{1}{R} \dot{P}(r_0) r_0 + \int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt \\
&= \int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt \quad \text{by (62)}
\end{aligned} \tag{65}$$

This means that the sectoral income can be split like this:

$$Y_0 = \underbrace{[P(r_0) - g(S_0)] r_0}_{\text{current cash flow}} - \underbrace{[P(r_0) - g(S_0)] r_0}_{\text{net investments}} + \underbrace{\int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt}_{\text{price change effects}}, \tag{66}$$

where the negative net investments equal the Hotelling rents and cancel out the value of production. Hence, the sectoral income can be interpreted as arising from the price change effects. The stock dependent extraction cost function has no explicit contributions to the sectoral income in this form. The current cash flow is canceled off by the sectoral net investment.

5.5 Technological improvement

Still consider the same settings as in the previous subsection 5.4. But now the extraction cost function is assumed to be the form

$$C(r_t, z_t) = r_t g(z_t) \quad (67)$$

Where z_t denotes the technological improvement. The effects of the remaining stock S is ignored temporarily.

Here we assume z_t is non-decreasing with respect to time t ($\dot{z}_t \geq 0$). To reflect the effect of technological improvement on reducing costs, in general, we furthermore assume a decreasing cost function with respect to z_t , which implies that

$$C'_z(r_t, z_t) = r_t g'(z_t) \leq 0 \implies g'(z_t) \leq 0 \quad (68)$$

since the extraction is always non-negative.

Suppose that the resource owner knows the effects of z_t on the cost function, and he has chosen the equilibrium path to extract. Through the similar approach to derive equation (59) we can obtain that the Hotelling rule means

$$\begin{aligned} [P(r_t) - g(z_t)] e^{-Rt} &= P(r_0) - g(z_0) = \text{constant or} \\ \dot{P}(r_0) - \dot{g}(z_t) &= R [P(r_t) - g(z_t)] \end{aligned} \quad (69)$$

From conditions (69) we know that interests on Ricardian rent or real cash flow now equal the real price change plus the reduced unit cost of the extraction due to the technological improvement at each point in time.

The real cash flow at each point in time is

$$P(r_t)r_t - C(r_t, z_t) = [P(r_t) - g(z_t)] r_t$$

Since we know

$$\begin{aligned}
& \frac{\partial}{\partial t} [P(r_t) - g(z_t)] r_t \\
&= [P(r_t) - g(z_t)] \dot{r}_t + [\dot{P}(r_t) - \dot{g}(z_t)] r_t \\
&= [P(r_t) - g(z_t)] \dot{r}_t + R [P(r_t) - g(z_t)] r_t
\end{aligned} \tag{70}$$

If we interpret $g(z_t)$ as a price of extraction services or the effect of technological improvement as an adjustment to a price of extraction services², the sectoral income at current time can be found out by expression (3)

$$\begin{aligned}
Y_0 &= [P(r_0) - g(z_0)] r_0 + \int_0^\infty e^{-Rt} [P(r_t) - g(z_t)] \dot{r}_t dt + \int_0^\infty e^{-Rt} [\dot{P}(r_t) - \dot{g}(z_t)] r_t dt \\
&= [P(r_0) - g(z_0)] r_0 + [P(r_0) - g(z_0)] \int_0^\infty \dot{r}_t dt + \int_0^\infty e^{-Rt} R [P(r_t) - g(z_t)] r_t dt \\
&= [P(r_0) - g(z_0)] r_0 + [P(r_0) - g(z_0)] (-r_0) + R [P(r_0) - g(z_0)] \int_0^\infty r_t dt \\
&= R [P(r_0) - g(z_0)] S_0
\end{aligned} \tag{71}$$

Similarly, this means that the sectoral income can be split like this:

$$Y_0 = \underbrace{[P(r_0) - g(z_0)] r_0}_{\text{current cash flow}} - \underbrace{[P(r_0) - g(z_0)] r_0}_{\text{net investments}} + \underbrace{R [P(r_0) - g(z_0)] S_0}_{\text{price change effects}}, \tag{72}$$

where the negative net investments equal the Hotelling rents and cancel out the value of production. Hence, the sectoral income can be interpreted as arising from the price change effects.

The income can also be interpreted as interests on the present value of the current resource stock no matter what will happen to the technological changes. In this simplified case, the sectoral income could be worked out without knowing any information about future variables.

²Assume production function (6) has the separable form as $r = F(a, z) = a \cdot h(z)$. Then the extraction cost equals $c(r, z) = Wa = r \cdot W/h(z)$. By comparing it with expression (67), it is easy to derive that $g(z) = W/h(z)$, where $h(z)$ can be thought of as the ratio of r to a . Therefore, it is natural to think of $g(z)$ as a price of extraction services, which declines over time due to the effect of technological improvement. Or the effect of technological improvement is thought of as an adjustment to a price of extraction services.

5.6 Technological improvement and stock dependent cost

Still consider the same settings as in the previous subsection 5.4. But now the extraction cost function is assumed to be the form

$$C(r_t, S_t, z_t) = r_t g(S_t, z_t) \quad (73)$$

The above assumptions related to the remaining stock and technological improvement still hold in this case. Obviously,

$$C'_r(r_t, z_t) = g(S_t, z_t) \quad (74)$$

Suppose that the resource owner knows the effect of S and z on the cost function, and he has chosen the equilibrium path to extract. Through the similar approach to derive equation (59) we can obtain that the extraction must go along the way that satisfies the Hotelling rule as

$$\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t = R[P(r_t) - g(S_t, z_t)]. \quad (75)$$

Then the changes of the remaining stock do not appear in the equation (75).

The real cash flow at each point in time

$$P(r_t)r_t - C(r_t, S_t, z_t) = [P(r_t) - g(S_t, z_t)]r_t$$

Since we know

$$\begin{aligned} & \frac{\partial}{\partial t} \{ [P(r_t) - g(S_t, z_t)] r_t \} \\ = & [P(r_t) - g(S_t, z_t)] \dot{r}_t + \frac{\partial [P(r_t) - g(S_t, z_t)]}{\partial t} r_t \\ = & \frac{\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t}{R} \dot{r}_t + \frac{1}{R} \frac{\partial [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t]}{\partial t} r_t \\ = & \frac{1}{R} \frac{\partial}{\partial t} \{ [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t] r_t \} \end{aligned} \quad (76)$$

If the effect of technological improvement is thought of as an adjustment to a price of

extraction services³, then by expression (3) The sectoral income at current time is then

$$\begin{aligned}
Y_0 &= [P(r_0) - g(S_0, z_0)] r_0 + \int_0^\infty e^{-Rt} [P(r_t) - g(S_t, z_t)] \dot{r}_t dt \\
&\quad + \int_0^\infty e^{-Rt} \frac{\partial [P(r_t) - g(S_t, z_t)]}{\partial t} r_t dt \\
&= [P(r_0) - g(S_0, z_0)] r_0 + \frac{1}{R} \int_0^\infty e^{-Rt} \frac{\partial}{\partial t} \{ [\dot{P}(r_0) - g'_z(S_t, z_t) \dot{z}_t] r_t \} dt \\
&= [P(r_0) - g(S_0, z_0)] r_0 \\
&\quad + \frac{1}{R} \left\{ e^{-Rt} [\dot{P}(r_0) - g'_z(S_t, z_t) \dot{z}_t] r_t \Big|_0^\infty - \int_0^\infty (-R) e^{-Rt} [\dot{P}(r_0) - g'_z(S_t, z_t) \dot{z}_t] r_t dt \right\} \\
&= \int_0^\infty e^{-Rt} [\dot{P}(r_0) - g'_z(S_t, z_t) \dot{z}_t] r_t dt \\
&= \int_0^\infty e^{-Rt} \dot{P}(r_t) r_t dt - \int_0^\infty e^{-Rt} g'_z(S_t, z_t) \dot{z}_t r_t dt
\end{aligned} \tag{77}$$

Similarly, this means that the sectoral income can be split like this:

$$Y_0 = \underbrace{[P(r_0) - g(S_0, z_0)] r_0}_{\text{current cash flow}} - \underbrace{[P(r_0) - g(S_0, z_0)] r_0}_{\text{net investments}} + \underbrace{\int_0^\infty e^{-Rt} [\dot{P}(r_0) - g'_z(S_t, z_t) \dot{z}_t] r_t dt}_{\text{price change effects}}, \tag{78}$$

where the negative net investments equal the Hotelling rents and cancel out the value of current cash flow. Hence, the sectoral income can be interpreted as arising from the price change effects.

Since in expression (77) $g'_z(S_t, z_t) \dot{z}_t \leq 0$, the income in this case is greater than the effects of petroleum price change. In this case, the technological improvement has positive effect on the income of petroleum extraction sector. Remember in the previous two subsections there are no explicit effects of the remaining stock S or the technological variable z on the sectoral income if in the cost function we neglect any one of the two factors S and z . But now both S and z appear in the expression of the sectoral income.

In order to estimate the sectoral income, we have to know the future petroleum prices and future technological improvement.

³Similarly as that in the previous case, if assume production function (6) has the separable form as $r = F(a, S, z) = a \cdot h(S, z)$. Then it is easy to derive that $g(S, z) = W/h(S, z)$. Here it is not suitable to think of $g(S, z)$ as a price of extraction services since it includes the effect of the remaining stock over time, which depends on the extraction rate. However, it still makes sense to think of the effect of technological improvement over time ($g'_z(S, z) \dot{z}$) as an adjustment to a price of extraction services.

5.7 Effects of time dependent real interest rate

Continue to consider the same settings as in the previous subsection 5.4. But now we release the assumption of constant real interest rate over time. Consider the model in the previous subsection 5.6.

Suppose that the resource owner knows the effect of S_t and z_t on cost function, and he has chosen the equilibrium path to extract. Through the similar approach to derive equation (59) we can obtain the Hotelling rule

$$\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t = R_t [P(r_t) - g(S_t, z_t)] \quad (79)$$

Now the Divisia consumer price index π_t is no longer equal to e^{-Rt} . Instead, we have

$$\pi_t = \exp\left(-\int_t^\infty R_\tau d\tau\right) \quad (80)$$

There is no effect on the expression of real cash flow and equation (76) is still satisfied after replacing R with R_t .

Similar to expression (77), the sectoral income at current time then becomes by expression (3)

$$\begin{aligned} Y_0 &= [P(r_0) - g(S_0, z_0)] r_0 + \int_0^\infty \frac{1}{R_t} \pi_t \frac{\partial}{\partial t} \left\{ [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t] r_t \right\} dt \\ &= [P(r_0) - g(S_0, z_0)] r_0 \\ &\quad + \left\{ \frac{1}{R_t} \pi_t [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t] r_t \Big|_0^\infty - \int_0^\infty (-\pi_t) [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t] r_t dt \right\} \\ &= \int_0^\infty \pi_t [\dot{P}(r_t) - g'_z(S_t, z_t)\dot{z}_t] r_t dt \\ &= \int_0^\infty \pi_t \dot{P}(r_t) r_t dt - \int_0^\infty \pi_t g'_z(S_t, z_t) \dot{z}_t r_t dt \end{aligned} \quad (81)$$

After introducing the time dependent real interest rate, we find that the expression for the sectoral income does not change essentially, just replace e^{-Rt} with real Divisia consumer price index π_t . However, given the future path of petroleum price, extraction and technological improvement, a decreasing real interest rate implies more income of petroleum sector in this case since $\pi_t > e^{-Rt}$ always holds. However, we get a different

conclusion in subsection 5.1.2, where we do not know whether the sectoral income will increase or not if the real interest rate is decreasing.

6 Partial models with exogenous price determination

This section emphasizes on the models where the market petroleum real price is determined exogenously. Firms in the sector are price taker and have full information on the exogenously given price path.

6.1 Constant unit extraction cost

First consider the simple case with constant unit extraction cost. Each unit of the extraction has the same cost any time.

$$C(r_t) = cr_t, \quad c \text{ is a positive constant}$$

Obviously, constant real price P implies to extract all at once if $P > c$ or to keep all in the ground if $P < c$.

Then let us assume a given price path over time that increases with a constant rate α , i.e. $P_t = P_0 e^{\alpha t}$. Does it mean zero income arising from the petroleum sector in this case if the current price is less than the constant unit cost, $P_0 < c$? If $\alpha \geq R$, there is no solution, since extraction will be postponed indefinitely. However, some income can be generated if $\alpha < R$ since one day in the future, the petroleum real price will exceed the constant cost and it will become more profitable to extract than to keep the petroleum in the ground. This case roughly reflects the long period before the extraction of underground petroleum with marginal cost higher than current market price.

Say at the future time m the present value of the profit or net price $(P_m - c)$ is the greatest over time. This can be got by maximizing $(P_0 e^{\alpha t} - c)e^{-Rt}$ and solving to find

the equilibrium time $t = m$. Then we know that P_m must satisfy

$$\begin{aligned}\frac{\alpha P_m}{P_m - c} &= R \iff \\ P_m - c &= \frac{\alpha P_m}{R} = \frac{\alpha P_0 e^{\alpha m}}{R},\end{aligned}\tag{82}$$

which states that at the time point m the marginal net return on the extraction is the largest and equal to R . All the stock S_0 will be extracted at the time point m . Thus, the income arising from the petroleum sector is

$$\begin{aligned}Y_0 &= Re^{-Rm}(P_m - c)S_0 \\ &= Re^{-Rm}\frac{\alpha P_0 e^{\alpha m}}{R}S_0 \quad \text{by (82)} \\ &= \alpha e^{-(R-\alpha)m}P_0 S_0 > 0\end{aligned}\tag{83}$$

This means that by expression (3) the sectoral income can be split like this:

$$Y_0 = \underbrace{0}_{\text{current cash flow}} + \underbrace{0}_{\text{net investments}} + \underbrace{\alpha e^{-(R-\alpha)m}P_0 S_0}_{\text{price change effects}},\tag{84}$$

since current extraction is zero. Then, the resource income can be interpreted as arising from price change effects.

6.2 Variable extraction costs w.r.t. extraction alone

The analysis in this subsection follows that in section 7 of Asheim and Wei (2006).

Consider a reservoir of petroleum. In this case, assume the cost is a function of extraction r_t alone, which then can be written as $C(r_t)$. The form of the cost function is supposed to be the same at each point in time. The real petroleum price is constant over time as P . This is the similar case described in Vincent et al. (1997, Section II), where they view constant consumption arising from the petroleum over time as "interest on a constant total capital stock equal to the sum of foreign investments and the capitalized value of oil resources".

Then the real cash flow at time t can be written as follows:

$$Pr_t - C(r_t) = (P - C'(r_t))r_t + (C'(r_t)r_t - C(r_t)).$$

Hotelling's rule tells us that

$$e^{-Rt}(P - C'(r_t)) = P - C'(r_0) = \text{constant} \quad (85)$$

Hence, by equation (85) the income at current time 0 of the reservoir can be written as

$$\begin{aligned} Y_0 &= \int_0^T Re^{-Rt}(Pr_t - C(r_t))dt \\ &= R(P - C'(r_0))S_0 + R \int_0^T e^{-Rt}(C'(r_t)r_t - C(r_t))dt. \end{aligned} \quad (86)$$

The first term is interests on the present value of future Hotelling's rent, while the second term is interest on the present value of future Ricardian rent.

Alternatively, we can use expression (3) to derive expressions for the income of a reservoir. Since P are assumed to be constant, we obtain

$$\begin{aligned} Y_0 &= Pr_0 - C(r_0) + \int_0^\infty e^{-Rt}(P - C'(r_t))\dot{r}_t dt \\ &= Pr_0 - C(r_0) + (P_0 - C'(r_0)) \int_0^\infty \dot{r}_t dt \quad \text{by (85),} \\ &= C'(r_0)r_0 - C(r_0) \quad \text{since } \lim_{t \rightarrow \infty} r_t = 0 \text{ implies } \int_0^\infty \dot{r}_t dt = -r(0). \end{aligned} \quad (87)$$

This means that the income of the reservoir can be split like this:

$$Y_0 = \underbrace{Pr_0 - C(r_0)}_{\text{current cash flow}} - \underbrace{[P_0 - C'(r_0)]r_0}_{\text{net investments}} + \underbrace{0}_{\text{price change effects}}, \quad (88)$$

since the real price keeps constant. Then, the resource income can be interpreted as current cash flow net of the net investments. The latter cancels off the current Hotelling rent. Hence, we arrive at the result that income of the reservoir – given the assumptions that we have made – equals current Ricardian rent.

Notice that there are actually two kinds of inputs in the production process as we mentioned in the section 3. They are the effort a , which is included in the analysis, and the ground, represented by the remaining resource stock S , which is ignored in the above calculation. If we suppose the production function $F(a, S)$ as constant returns to scale, then multiply with $C'_r(r)$ on both sides of (9) and rearrange,

$$\begin{aligned}
C''(r)F'_S S &= C''(r) [r - F'_a a] \\
&= C''(r) \left[r - F'_a(a, S) \frac{C(r, S)}{w} \right] \quad \text{by (16)} \\
&= C''(r)r - C(r) \quad \text{by (19) and (16)}
\end{aligned} \tag{89}$$

This is exactly the current Ricardian rent. Then we can interpret that the income of a reservoir is equal to the productivity of the ground input evaluated at the resource price net of the Hotelling rent.

It might be helpful to explain more on the Ricardian rent. It is principally very different from the Hotelling rent in the theory of exhaustible resource. One necessary premise for the existence of the Ricardian rent is that an input in the production consists of heterogeneous quality units. Following the Ricardian view (Ricardo, 1821), given that the best quality units of the input are put into production first, as demand increases lower quality units of the input is brought into production. Assuming that the market price has to equal unit cost of the marginal product, the price will exceed costs of all the output produced with more productive units of the input. Thus, the latter will earn Ricardian rent. Then, the Ricardian rent earned here is produced due to the heterogeneous units of the ground input. More Ricardian rent is generated with more extraction rate. Notice that this is only correct within each point in time since here expression (9) is invoked. And the difference of ground input quality (or the remaining stock) at the beginning of each point in time does not have effect on the extraction cost and then is irrelevant to the Ricardian rent within each point in time. In our cases, the level of Ricardian rent tends to decline over time since the marginal cost becomes lower and lower along with less and less extraction rate.

The Ricardian rent here is quantitatively associated with the extraction at each point in time r_t . If we take the Ricardian rent as one part of the cost, the unit extraction

cost at a given time t becomes constant as $C''(r_t)$, which implies that the cost function has the property of constant unit cost at each time point t . Note also that due to the variable extraction rate over time, the virtual unit cost $C''(r_t)$ is various over time. Then in the analysis, we always assume constant unit cost function within each point in time. Undoubtedly, we have to remember Ricardian rent on the resource is in fact one part of the income arising from the petroleum sector, not the real extraction cost.

6.3 Stock dependent cost

We still assume the same settings as in the previous case (6.2). But now the cost of the reservoir is a function with respect to not only the extraction rate, but also the remaining stock, which then can be written as $C(r_t, S_t)$. Then the real cash flow at time t can be written as follows:

$$Pr_t - C(r_t, S_t).$$

By expression (59), the Hotelling rule tells us that

$$R[P - C'_r(r_t, S_t)] = -C'_S(r_t, S_t) - \dot{C}'_r(r_t, S_t) \quad (90)$$

We also know that

$$\begin{aligned} \frac{\partial}{\partial t} [e^{-Rt} (P - C'_r) r_t] &= -Re^{-Rt} (P - C'_r) r_t - e^{-Rt} \dot{C}'_r r_t \\ &\quad + e^{-Rt} (P - C'_r) \dot{r}_t. \end{aligned} \quad (91)$$

Integrate on both sides of (91) and rearrange

$$\begin{aligned} &\int_0^\infty Re^{-Rt} (P - C'_r) r_t dt + \int_0^\infty e^{-Rt} \dot{C}'_r r_t dt \\ &= -[e^{-Rt} (P - C'_r) r_t] \Big|_0^\infty + \int_0^\infty e^{-Rt} (P - C'_r) \dot{r}_t dt \\ &= [P - C'_r(r_0, S_0)] r_0 + \int_0^\infty e^{-Rt} (P - C'_r) \dot{r}_t dt \end{aligned} \quad (92)$$

Then, we use expression (3) to derive the income of a reservoir

$$\begin{aligned}
Y_0 &= Pr_0 - C(r_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{r}_t dt - \int_0^\infty e^{-Rt} C'_S(r_t, S_t) \dot{S}_t dt \\
&= Pr_0 - C(r_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{r}_t dt + \int_0^\infty e^{-Rt} C'_S(r_t, S_t) r_t dt \\
&= Pr_0 - C(r_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{r}_t dt \\
&\quad - \left[\int_0^\infty e^{-Rt} R [P - C'_r(r_t, S_t)] r_t dt + \int_0^\infty e^{-Rt} \dot{C}'_r(r_t, S_t) r_t dt \right] \quad \text{by (90)} \\
&= Pr_0 - C(r_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{r}_t dt \\
&\quad - [P - C'_r(r_0, S_0)] r_0 - \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{r}_t dt \quad \text{by (92)} \\
&= C'_r(r_0, S_0) r_0 - C(r_0, S_0)
\end{aligned} \tag{93}$$

Similarly this means that the income of the reservoir can be split like this:

$$Y_0 = \underbrace{Pr_0 - C(r_0, S_0)}_{\text{current cash flow}} - \underbrace{[P - C'_r(r_0, S_0)] r_0}_{\text{net investments}} + \underbrace{0}_{\text{price change effects}},$$

since the real price keeps constant. Then, the resource income can be interpreted as current cash flow net of the net investments. The latter cancels off the current Hotelling rent. Hence, we arrive at the result that the income of the reservoir – given the assumptions that we have made – equals current Ricardian rent.

If we suppose the production function $F(a, S)$ as constant returns to scale within each point in time, then multiply with $C'_r(r, S)$ on both sides of (9) and rearrange,

$$\begin{aligned}
C'_r(r, S) F'_S S &= C'_r(r, S) [r - F'_a a] \\
&= C'_r(r, S) \left[r - F'_a(a, S) \frac{C(r, S)}{w} \right] \quad \text{by (16)} \\
&= C'_r(r, S) r - C(r) \quad \text{by (19) and (16)}
\end{aligned}$$

This is exactly the current Ricardian rent. Then we can interpret that the income of a reservoir is equal to the productivity of the ground input evaluated at the resource price net of the Hotelling rent.

6.4 Exogenously determined extraction

In reality, the petroleum market is far from the competitive structure. There are a lot of factors that affect the extraction plan. One is the uncertainty. No one can exactly predict the future so that it is almost impossible to find out the equilibrium extraction path. Another one is the limit of the technology. The common technology requires that much of the investment occurs up-front so that after a reservoir is developed, the extraction will be processed as soon as possible no matter what the price path is. Sometimes the firms can not extract as much as they can due to the current technology, in particular for a declining reservoir.

Then to a large extent, the extraction rate of firms does not rely on the petroleum price level. In this section we will consider the cases with exogenous extraction path when petroleum price path is given. Since the cost variation caused by the exogenous technological improvement z_t can be thought of as the price of extraction services, we assume constant technological variable z_t over time to eliminate the price effects from the technological improvement.

6.4.1 Constant unit extraction cost

Even though the extraction rate varies greatly over time, there are three basic possibilities, increasing, constant, and decreasing. In order to focus on the conceptual aspects, here we only consider the cases with constant changing rates for the extraction rate over time.

Assume the exogenously determined extraction path of a firm changes at a constant rate β over time, that is to say,

$$r_t = r_0 e^{\beta t}, \tag{94}$$

where $\beta > 0$ implies increasing extraction rate, $\beta < 0$ decreasing extraction rate, and $\beta = 0$ constant extraction rate over time.

The initial resource stock is supposed to be given as S_0 . Suppose at the point in time T the resource is exhausted, then along with the given extraction path, we must have the

relations as follows

$$\begin{aligned} \int_0^T r_t dt &= \int_0^T r_0 e^{\beta t} dt = S_0 \iff \\ &\begin{cases} Tr_0 = S_0 & \text{if } \beta = 0 \\ \frac{e^{\beta T} - 1}{\beta} r_0 = S_0 & \text{otherwise} \end{cases} \end{aligned} \quad (95)$$

Since the initial stock S_0 and the extraction path are given in advance, the period with positive extraction rate T can be derived from the relations (95).

For simplicity, the real interest rate is assumed to be a positive constant over time as R and the petroleum real price over time is assumed constant as P . This crosses out the price change effects on the sectoral income.

As the first step to analyze the cases with exogenously determined extraction path, we suppose the unit extraction cost is constant over time as c , independent on the extraction rate or any other variables.

Through expression (2), the sectoral income can be calculated,

$$\begin{aligned} Y_0 &= \int_0^T R e^{-Rt} (P - c) r_t dt \\ &= \int_0^T R e^{-Rt} (P - c) r_0 e^{\beta t} dt \quad \text{by (94)} \\ &= R (P - c) r_0 \int_0^T e^{-(R-\beta)t} dt \\ &= \begin{cases} R (P - c) r_0 T & \text{if } \beta = R \\ \frac{R}{R-\beta} (P - c) r_0 [1 - e^{-(R-\beta)T}] & \text{otherwise} \end{cases} \end{aligned} \quad (96)$$

Then by expression (3), the income in (96) can be split into three terms

$$\begin{aligned} Y_0 &= \underbrace{(P - c) r_0}_{\text{current cash flow}} - \underbrace{(P - c) r_0 (1 - RT)}_{\text{net investments}} + \underbrace{0}_{\text{price change effects}}, \quad \text{if } \beta = R \\ &= \underbrace{(P - c) r_0}_{\text{current cash flow}} - \underbrace{\frac{R (P - c) r_0}{R - \beta} \left[e^{-(R-\beta)T} - \frac{\beta}{R} \right]}_{\text{net investments}} + \underbrace{0}_{\text{price change effects}}, \quad \text{otherwise} \end{aligned} \quad (97)$$

where the price change effect is zero since no real price change at all. The sectoral income then is equal to the current cash flow plus the net investments. It is interesting to notice

that the net investments might be positive, which requires

$$\left\{ \begin{array}{ll} RT > 1 \implies T > 1/R & \text{if } \beta = R \\ \frac{\beta}{R} > e^{-(R-\beta)T} \implies T > \frac{\ln R - \ln \beta}{R-\beta} & \text{if } 0 < \beta < R \\ \frac{\beta}{R} < e^{-(R-\beta)T} \implies T < \frac{\ln \beta - \ln R}{\beta-R} & \text{if } \beta > R \end{array} \right. . \quad (98)$$

Based on expression (98), if the exogenous extraction rate increases at a rate higher than the interest rate, then the net investments is positive only if the initial stock is so small that the extraction can only last for a certain short period. On the other hand, if the exogenous extraction rate increases at a rate no more than the interest rate, then the positive net investments can be obtained only if the initial stock is so large that the extraction can last for a rather long period.

However, we can not conclude that the sectoral income becomes higher when we switch from one extraction path with negative net investments to another with positive net investments. This is because during the adjustment, the current cash flow is changing too.

6.4.2 Variable cost w.r.t. extraction alone

We still assume the same settings as in the previous case (6.4.1). But now the cost of the firm varies with respect to the extraction alone as $C(r_t)$ and we assume the extraction path is exogenously determined as $\{\gamma_t\}_{t=0}^T$. The new remaining stock series is denoted by $\{\varsigma_t\}_{t=0}^T$. As shown in subsection 6.2, no matter what the extraction path is, the income equals current Ricardian rent as long as the petroleum price and marginal cost w.r.t. extraction rate go along with that in the equilibrium path. Then by expression (3) we

can obtain the income of the firm,

$$\begin{aligned}
Y_0 &= P\gamma_0 - C(\gamma_0) + \int_0^\infty e^{-Rt} (P - C'(\gamma_t)) \dot{\gamma}_t dt \\
&= P\gamma_0 - C(\gamma_0) + \int_0^\infty e^{-Rt} (P - C'(r_t)) \dot{\gamma}_t dt \\
&\quad + \int_0^\infty e^{-Rt} [C'(r_t) - C'(\gamma_t)] \dot{\gamma}_t dt \\
&= P\gamma_0 - C(\gamma_0) + (P - C'(r_0)) \int_0^\infty \dot{\gamma}_t dt \\
&\quad + \int_0^\infty e^{-Rt} [C'(r_t) - C'(\gamma_t)] \dot{\gamma}_t dt \quad \text{by Hotelling's rule,} \\
&= C'(r_0)\gamma_0 - C(\gamma_0) + \int_0^\infty e^{-Rt} [C'(r_t) - C'(\gamma_t)] \dot{\gamma}_t dt. \tag{99}
\end{aligned}$$

Hence, we arrive at the result that the income of a firm equals current Ricardian rent plus an adjustment term which comes from the difference between future equilibrium marginal cost and the firm individual's marginal cost. It can be shown that the adjustment term is negative⁴. Then the current Ricardian rent represents the upper limit of the income.

If the extraction path is exogenously determined such that the producer receives the same present value of the difference between future equilibrium marginal cost and the firm individual's marginal cost,

$$e^{-Rt} [C'(r_t) - C'(\gamma_t)] = C'(r_0) - C'(\gamma_0) = \text{constant},$$

for all $t \geq 0$. then expression (99) is simplified as

$$\begin{aligned}
Y_0 &= C'(r_0)\gamma_0 - C(\gamma_0) + [C'(r_0) - C'(\gamma_0)] \int_0^\infty \dot{\gamma}_t dt \\
&= C'(r_0)\gamma_0 - C(\gamma_0) - [C'(r_0) - C'(\gamma_0)] \gamma_0 \\
&= C'(\gamma_0)\gamma_0 - C(\gamma_0), \tag{100}
\end{aligned}$$

If we include the remaining stock S in the production function and suppose the production function $F(a, S)$ as constant returns to scale, then expression (89) is still satisfied

⁴First, the individual marginal cost must be less than equilibrium marginal cost, or the firm suffers loss. Then $C'(r_t) \geq C'(\gamma_t)$. we can find a small positive number $\delta \leq e^{-Rt} [C'(r_t) - C'(\gamma_t)]$ for all $t \geq 0$. If the adjustment term is positive, then we have that $\int_0^\infty e^{-Rt} [C'(r_t) - C'(\gamma_t)] \dot{\gamma}_t dt \leq \int_0^\infty \delta \dot{\gamma}_t dt = -\delta\gamma_0 \leq 0$, which is contradiction.

in this case. The right hand of expression (89) is exactly the same as the income in (100). Then we can interpret that the income of the firm is equal to the productivity of the ground input evaluated at its individual marginal cost.

6.4.3 Stock dependent cost

In this subsection, the assumptions in the previous case (6.4.2) holds except the follows. Now the cost is a function with respect to not only the extraction rate r , but also the remaining stock S . The function is denoted as $C(r_t, S_t)$. Then the Hotelling rule as expression (90) is still satisfied for equilibrium marginal cost w.r.t. extraction rate and the remaining stock. If the firm produces along with the equilibrium extraction path, $\{r_t\}_{t=0}^T$ as that in subsection 6.3, then its income is the current Ricardian rent. However, now we assume the extraction path is exogenously determined other than the equilibrium one as $\{\gamma_t\}_{t=0}^T$. The new remaining stock series is denoted by $\{\varsigma_t\}_{t=0}^T$. As shown in subsection 6.3, no matter what the extraction path is, the income equals the current Ricardian rent as long as the petroleum price and marginal cost w.r.t. extraction rate and the remaining stock go along with that in the equilibrium path. Then by expression (3) we can obtain the income of the firm,

$$\begin{aligned}
Y_0 &= P\gamma_0 - C(\gamma_0, S_0) + \int_0^\infty e^{-Rt} (P - C'_r(\gamma_t, \varsigma_t)) \dot{\gamma}_t dt - \int_0^\infty e^{-Rt} C'_S(\gamma_t, \varsigma_t) \dot{\varsigma}_t dt \\
&= P\gamma_0 - C(\gamma_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t) + C'_r(r_t, S_t) - C'_r(\gamma_t, \varsigma_t)] \dot{\gamma}_t dt \\
&\quad - \int_0^\infty e^{-Rt} [C'_S(r_t, S_t) + C'_S(\gamma_t, \varsigma_t) - C'_S(r_t, S_t)] \dot{\varsigma}_t dt \\
&= P\gamma_0 - C(\gamma_0, S_0) + \int_0^\infty e^{-Rt} [P - C'_r(r_t, S_t)] \dot{\gamma}_t dt + \int_0^\infty e^{-Rt} C'_S(r_t, S_t) \gamma_t dt \\
&\quad + \int_0^\infty e^{-Rt} [C'_r(r_t, S_t) - C'_r(\gamma_t, \varsigma_t)] \dot{\gamma}_t dt + \int_0^\infty e^{-Rt} [C'_S(\gamma_t, \varsigma_t) - C'_S(r_t, S_t)] \gamma_t dt \\
&= C'_r(r_0, S_0) \gamma_0 - C(\gamma_0, S_0) \\
&\quad + \int_0^\infty e^{-Rt} [C'_r(r_t, S_t) - C'_r(\gamma_t, \varsigma_t)] \dot{\gamma}_t dt + \int_0^\infty e^{-Rt} [C'_S(\gamma_t, \varsigma_t) - C'_S(r_t, S_t)] \gamma_t dt.
\end{aligned} \tag{101}$$

Hence, we arrive at the result that the income of a firm equals current Ricardian rent plus two adjustment terms. The two terms come from the difference between future

equilibrium marginal cost and the firm individual's marginal cost w.r.t. extraction rate and the remaining stock respectively.

Similarly as in the previous subsection 6.4.2, it can be shown that the first adjustment term is negative if $\gamma_t < r_t$. At the same time, if $\gamma_t < r_t$ always holds, then we always have $\varsigma_t > S_t$ and thus the second adjustment term is also negative since in general the cost tends to increase along with the reducing remaining stock, which implies $C'_S(r_t, S_t) < 0$. Then the second adjustment term is also negative. Therefore, the current Ricardian rent still represents the upper limit of the income. More share of it has to be deducted when compared with the previous case in subsection 6.4.2, where we ignore the remaining stock in the cost function.

If the extraction path is exogenously determined such that the producer receives the same present value of the difference between future equilibrium marginal cost and the firm individual's marginal cost,

$$e^{-Rt} [C'_r(r_t, S_t) - C'_r(\gamma_t, \varsigma_t)] = C'_r(r_0, S_0) - C'_r(\gamma_0, S_0) = \text{constant},$$

for all $t \geq 0$, furthermore, if the present value of the difference between future equilibrium marginal cost and individual marginal cost w.r.t. the remaining stock keeps constant over time,

$$e^{-Rt} [C'_S(\gamma_t, \varsigma_t) - C'_S(r_t, S_t)] = C'_S(\gamma_0, S_0) - C'_S(r_0, S_0) = \text{constant},$$

for all $t \geq 0$, then expression (101) is simplified as

$$\begin{aligned} Y_0 &= C'_r(r_0, S_0) - C(\gamma_0, S_0) + [C'_r(r_0, S_0) - C'_r(\gamma_0, S_0)] \int_0^\infty \dot{r}_t dt \\ &\quad + [C'_S(\gamma_0, S_0) - C'_S(r_0, S_0)] \int_0^\infty r_t dt \\ &= C'_r(r_0, S_0) - C(\gamma_0, S_0) - [C'_r(r_0, S_0) - C'_r(\gamma_0, S_0)] r_0 \\ &\quad + [C'_S(\gamma_0, S_0) - C'_S(r_0, S_0)] S_0 \\ &= C'_r(\gamma_0, S_0) r_0 - C(r_0, S_0) + [C'_S(\gamma_0, S_0) - C'_S(r_0, S_0)] S_0, \end{aligned} \quad (102)$$

If we suppose the production function $F(a, S)$ as constant returns to scale, then we

have the relations similar to expression (89) as follows,

$$\begin{aligned}
C'_r(\gamma, S)F'_S S &= C'_r(\gamma, S) [r - F'_a a] \quad \text{by (9)} \\
&= C'_r(\gamma, S) \left[r - F'_a(a, S) \frac{C(\gamma, S)}{w} \right] \quad \text{by (16)} \\
&= C'_r(\gamma, S)r - C(\gamma, S) \quad \text{by (19) and (16)}.
\end{aligned}$$

This is exactly the same as the first two terms in the right hand of (102). Then we can interpret that the income of the firm is equal to the productivity of the ground input evaluated at its individual marginal cost net of the difference between equilibrium marginal cost and individual marginal cost w.r.t the current remaining stock.

7 Concluding remarks

Through applying the new developed concept of sectoral income, the thesis discussed income arising from the petroleum extraction sector in various theoretical models. Income arising from the petroleum sector comes from the sectoral current and future cash flow, which can be classified as two types of resource rents: Hotelling's rent and Ricardian rent. However, in all cases we discussed, Hotelling's rent is not included in the sectoral income. In general, it is canceled off by part (or whole) negative sectoral net investments. On the other hand, Ricardian rent is one part of the sectoral income. But in the cases of exogenously determined extraction, part of Ricardian rent is deducted from the sectoral income. As the thesis displays, Ricardian rent is generated due to the heterogenous "quality" of the ground input during the extraction activity. The ground input is quantified as the remaining stock in the production and cost function.

In a Dasgupta-Heal-Solow model, the resource income equals the price change effects since the net investment effects cancel off the value of current production. The resource income is less than interests on the petroleum wealth since the real interest rate goes down over time.

Then in the partial equilibrium analysis, we considered four main factors that have effects on the sectoral income: petroleum real price, real interest rate, the remaining stock at a time point, and the technological improvement. Generally higher real prices

and technological improvement implies higher sectoral income. Less remaining stock implies higher extraction cost and then less sectoral income. However, the effects of the real interest rate depends on the specific cases. For example, when the extraction only lasts for a certain period, the decreasing real interest rate might imply more income arising from the petroleum sector as the analysis in subsection 5.7 by increasing the initial petroleum price.

In the partial equilibrium analysis, we classify partial models as two types: one with endogenous price determination and another with exogenous price determination. The thesis discussed the cases of competitive profit-maximizing firms, where, in general, the sectoral income equals the price change effects. The case of the monopolist is also discussed. The monopolist's income does not equal the price change effects. In fact, we can not split the net investments and price change effects explicitly since the price change is realized by the extraction rate adjustment. In section 6, the models with exogenous price determination are analyzed. If the petroleum real price is constant over time, the sectoral income is current Ricardian rent if the equilibrium extraction path is followed. However, if the extraction path are exogenously determined other than in equilibrium, current Ricardian rent is in general the upper limit of the sectoral income. the sectoral income includes an adjustment term that comes from the difference between future equilibrium and individual marginal cost.

The discussion in the thesis assumes that all the information is known at each point in time. This rules out the effects of uncertainty. However, uncertainty has to be carefully considered if we want to apply the theory into practice.

References

- Asheim, G.B. and Wei, T. (2006), Sectorial income, mimeo.
- Dasgupta, P.S. and Heal, G.M. (1974), The optimal depletion of exhaustible resources, *Review of Economic Studies* (Symposium), 3–28.
- Dasgupta, P.S. and Heal, G.M. (1979), *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge, UK.

- Farzin, Y.H. (1992), The time path of scarcity rent in the theory of exhaustible resources, *The Economic Journal*, 102 (July 1992), pp. 813–830.
- Hanson, D.A. (1980), Increasing extraction costs and resource prices: some further results, *Bell Journal of Economics*, vol. 11 (Spring), pp. 335–42.
- Hartwick, M.J. and Olewiler, D.N. (1998), Nonrenewable resource use: the theory of depletion, *The Economics of Natural Resource Use*, chapter. 8, pp. 288–9.
- Heal, G.M. (1976), The relationship between price and extraction cost for a resource with a backstop technology, *Bell Journal of Economics*, vol. 7 (Autumn), pp. 371–8.
- Hicks, J.R. (1939), *Value and Capital: An Inquiry into some Fundamental Principles of Economic Theory*. Oxford University Press.
- Hotelling, H. (1931), The economics of exhaustible resources, *Journal of Political Economy* 39, pp. 137–175.
- Ricardo, D. (1821), On the principles of political economy and taxation, 3rd Ed., 1821 from *The Works and Correspondence of David Ricardo*, edited by Piero Sraffa with the collaboration of Dobb, M.H., Cambridge University Press, 1951.
- Samuelson, P. (1961), The evaluation of ‘social income’: Capital formation and wealth, in F.A. Lutz and D.C. Hague (eds.), *The Theory of Capital*, St. Martin’s Press, New York, 32–57.
- Sefton, J.A. and Weale, M.R. (2005), The concept of income in a general equilibrium. Forthcoming in *Review of Economic Studies*.
- Solow, R.M. (1974), Intergenerational equity and exhaustible resources, *Review of Economic Studies* (Symposium), 29–45.
- Solow, R.M. and Wan, F.Y. (1976), Extraction costs in the theory of exhaustible resources, *Bell Journal of Economics*, vol. 7 (Autumn), pp. 359–70.
- Usher, D. (1994), Income and the Hamiltonian, *Review of Income and Wealth* 40, 123–41.

Vincent, J.R. Panayotou, T.P. and Hartwick, J.M. (1997), Resource depletion and sustainability in small open economies, *Journal of Environmental Economics and Management* 33, pp.274–286 (1997).